CLOCK RATE COMPARISONS BY LONG BASELINE INTERFEROMETRY

W. H. Cannon, R. B. Langley, W. T. Petrachenko
Department of Physics and CRESS, York University, Toronto, Canada.

ABSTRACT

We report a comparison of the determination of the frequency difference between remotely sited frequency standards by: (a) the technique of long baseline interferometry (b) the use of Loran-C/Time Service transmissions. In the long baseline interferometer technique the frequency difference between the local oscillators at each antenna site was modelled as a polynomial in time whose coefficients were estimated from the interferometer fringe frequencies simultaneously with other parameters of the interferometer.

The agreement between the two techniques ranged from 5 parts in 10^{13} r.m.s. for a 5 day continuous interval in March 1973 to 6 parts in 10^{12} r.m.s. for a 2 day continuous interval in June 1973. A number of experimental difficulties degraded the results of the June 1973 experiment and the former figure of 5 parts in 10^{13} is regarded as being representative of the expected agreement between the two techniques.

I INTRODUCTION

We report the results of some relative clock rate measurements accomplished by remote comparison using the technique of long baseline interferometry (LBI). The interferometer was made up of the 25 m. antenna at Chilbolton, England now known as the Chilbolton Observatory (CRO) operated by the Appleton Laboratory facility of the U. K. Science Research Council, and the 46 m. antenna of the Algonquin Radio Observatory (ARO) at Algonquin Park, Ontario, Canada operated by the National Research Council of Canada. The interferometer which has a baseline of 5265 km. was operated at 10,680 MHz (2.8 cm. wavelength). The antenna sites were equipped with cooled parametric amplifiers, local oscillators phase locked to Hewlett Packard H-10A atomic hydrogen masers, and IVC 800 series video tape recorders recording an IF bandwidth of about 5 MHz on standard one inch wide video tape.

The three experiments in March, May and June of 1973 were originally intended only to reveal the structure of a number of extra galactic radio sources (Legg et al., 1973; Legg et al., 1977). However the structure and operation of the interferometer were of a sufficiently high quality that the interferometer output, particularly the fringe frequencies, held the potential of yielding useful geodetic and astrometric results as well (Cannon et al., 1977).

The model used in the geodetic and astrometric analysis of the LBI fringe frequencies included a polynomial in time representing the relative frequency offset of the two on-site local oscillators, the coefficients of which were to be estimated simultaneously with the geodetic and astrometric parameters. In the results of the LBI parameter estimation procedure the coefficients of the local oscillator polynomial were correlated (in certain cases with correlation coefficients in excess of 0.90) with the solutions for the other parameters of the model. In order to assess the significance of these correlations and to test the reliability of the LBI geodetic and astrometric solutions it was decided to attempt an independent estimate, using Loran-C transmissions, of the local oscillator frequency offset for comparison with the LBI solution.

The data necessary to make such a comparison were available from the station logs for the March and June experiments. For the most part this data consisted of a record at each site, maintained throughout the experiment, of the drift of the "station clock" (driven by the hydrogen maser in each case) relative to the time of arrival of a given Loran-C pulse. The "Daily Phase Values" published in USNO Time Service Publication Series 4 would allow a determination of the relative drift of the two Loran-C pulses and thus a determination of the relative drift of the two "station clocks".

A schematic diagram depicting that portion of the experimental arrangement relevant to this study is shown in FIGURE 1. The frequency standard for the "station clock" at ARO is a hydrogen maser H.P.-10A Ser. 9 maintaining a time scale designated UTC[ARO-H9] while at CRO the frequency standard for the "station clock" is a hydrogen maser H.P.-10A Ser. 11 maintaining a time scale designated UTC[CRO-H11]. The Loran-C pulses received at ARO and CRO during these experiments were emitted from the Nantucket slave station (designated 9930Y) of the U.S. East Coast chain and from the Sylt slave station (designated 7970W) of the Norwegian Sea chain respectively. The reception of the Loran-C pulses at ARO and CRO is used to materialize a time scale designated UTC Loran-C [ARO 9930Y] and UTC Loran-C[CRO 7970W] respectively.

The correlator for the interferometer used in these experiments was an analogue device for which the amplitude of a zero frequency fringe becomes indeterminate. The interferometer was also operating in a double side band mode which, while increasing the signal-to-noise ratio of the correlation function has the disadvantage of being unable to distinguish positive and negative fringe frequencies. In an effort to avoid experimental difficulties arising as a result of these two

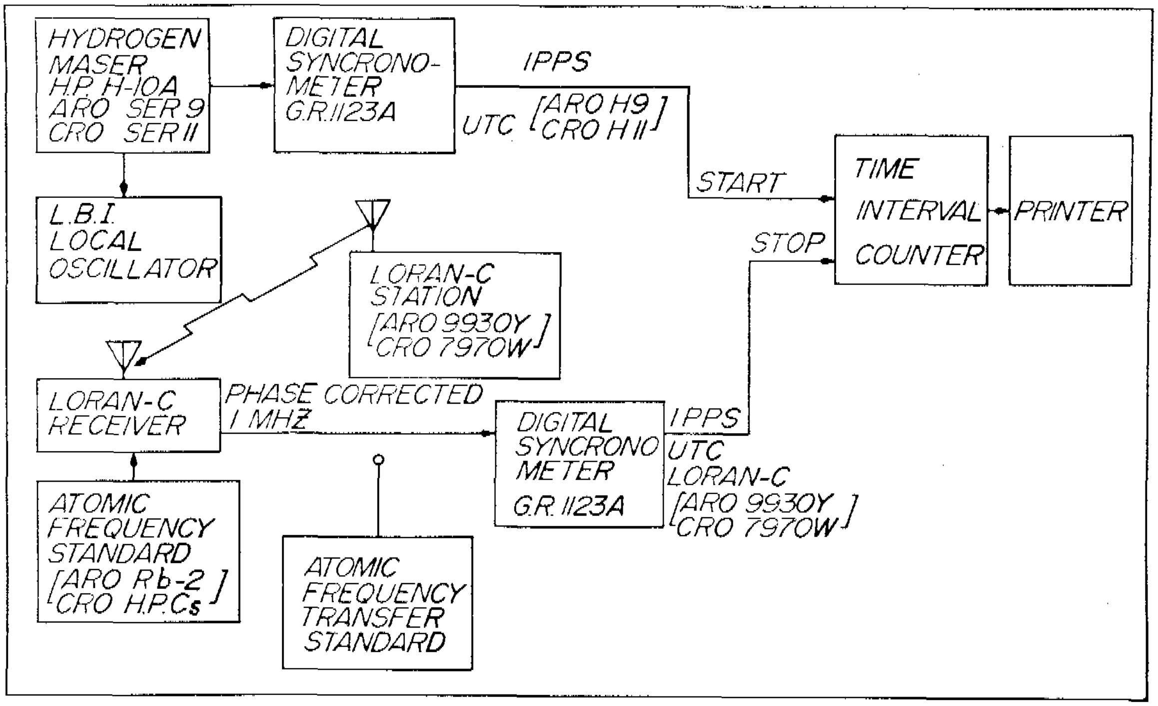

FIGURE 1. Schematic of experimental arrangement for comparing LBI local oscillators.

factors it was decided to deliberately offset the local oscillator at ARO from the local oscillator at CRO by a fixed frequency increment. This "nominal local oscillator offset" was 300 mHz in the March 1973 experiment and 500 mHz in the June 1973 experiment.

II MARCH 1973 EXPERIMENTAL RESULTS

The analysis of the experimental data for March 1973 proceeded as follows.

Defining $\tau_{ARO}(t)$ as

$$\tau_{ARO}(t) = \text{UTC[ARO H9]} - \text{UTC Loran-C[ARO 9930Y]} \tag{1}$$

and $\tau_{CRO}(t)$ as

$$\tau_{CRO}(t) = \text{UTC[CRO H11]} - \text{UTC Loran-C[CRO 7970W]} \tag{2}$$

and defining $\tau_{9930}(t)$ as

$$\tau_{9930}(t) = \text{UTC[USNO-M.C.]} - \text{UTC[Loran-C 9930]} \tag{3}$$

and $\tau_{7970}(t)$ as

$$\tau_{7970}(t) = \mathrm{UTC[USNO\ M.C.]} - \mathrm{UTC[LORAN\text{-}C\ 7970]} \tag{4}$$

then the drift of the station clocks relative to each other is $\Delta\tau_{ARO/CRO}(t)$ where

$$\Delta\tau_{ARO/CRO}(t) = [\tau_{ARO}(t) - \tau_{9930}(t)]-[\tau_{CRO}(t)-\tau_{7970}(t)] \tag{5}$$

The data for the determination of $\tau_{ARO}(t)$ and $\tau_{CRO}(t)$ were logged during the experiment of March 1973 at each antenna site and are shown together with a maximum likelihood polynomial fit for ARO and CRO in FIGURES 2a and 2b respectively. The error bars shown are $\pm$ 0.2 μs which we have taken as a measure of the standard deviation of a single determination of $\tau_{ARO}(t)$, $\tau_{CRO}(t)$. The order of the polynomial (which in this case is 3rd order) was chosen as the lowest order polynomial from which a further increase in order produced no significant reduction of variance.

The data for the determination of $\tau_{9930}(t)$ and $\tau_{7970}(t)$ are published as "Daily Phase Values and Time Differences" in USNO Time Service Publication Series 4 (USNO Time Service Publications 1973). In determining $\tau_{9930}(t)$ and $\tau_{7970}(t)$ from the "Daily Phase Values" for March 1973 maximum likelihood first order polynomials were fitted to the U. S. East Coast Chain data for February 14, 1973 to May 30, 1973 inclusive to give $\tau_{9930}(t)$ and to the Norwegian Sea Chain data for January 1, 1973 to June 4, 1973 inclusive to give $\tau_{7970}(t)$.

The local oscillator frequency offset between ARO and CRO is $\Delta f_o(t)$ where

$$\Delta f_o(t) = f_{o_{ARO}}(t) - f_{o_{CRO}}(t) \tag{6}$$

and where $f_{o_{ARO}}, f_{o_{CRO}}$ are the local oscillator frequencies, nominally 10,680 MHz, at ARO and CRO respectively. The local oscillator frequency offset as determined by the Loran-C/USNO M.C. data is $\Delta f_{o_{Loran}}(t)$ where

$$\Delta f_{o_{Loran}}(t) = f_o \cdot \frac{d}{dt}[\Delta\tau_{ARO/CRO}(t)] \tag{7}$$

and where f_o is the nominal local oscillator frequency.
The local oscillator frequency offset as determined by inversion of the LBI fringe frequency data is $\Delta f_{o_{LBI}}(t)$.

FIGURE 3 shows a comparison of $\Delta f_{o_{Loran}}(t)$ and $\Delta f_{o_{LBI}}(t)$. The LBI fringe frequency model for $\Delta f_{o_{LBI}}(t)$ includes small relativistic effects

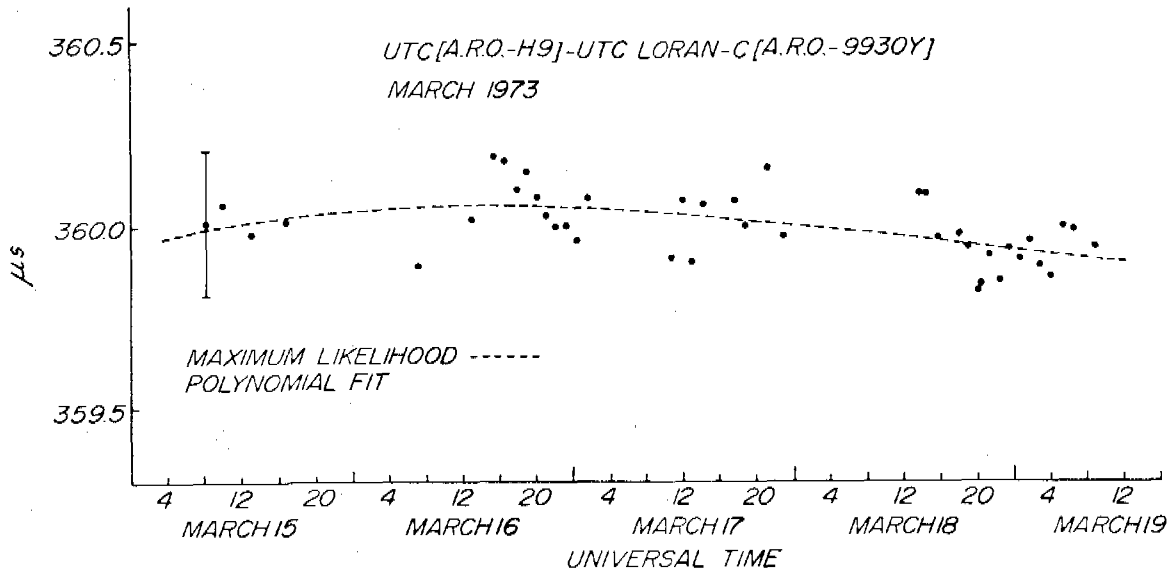

FIGURE 2a

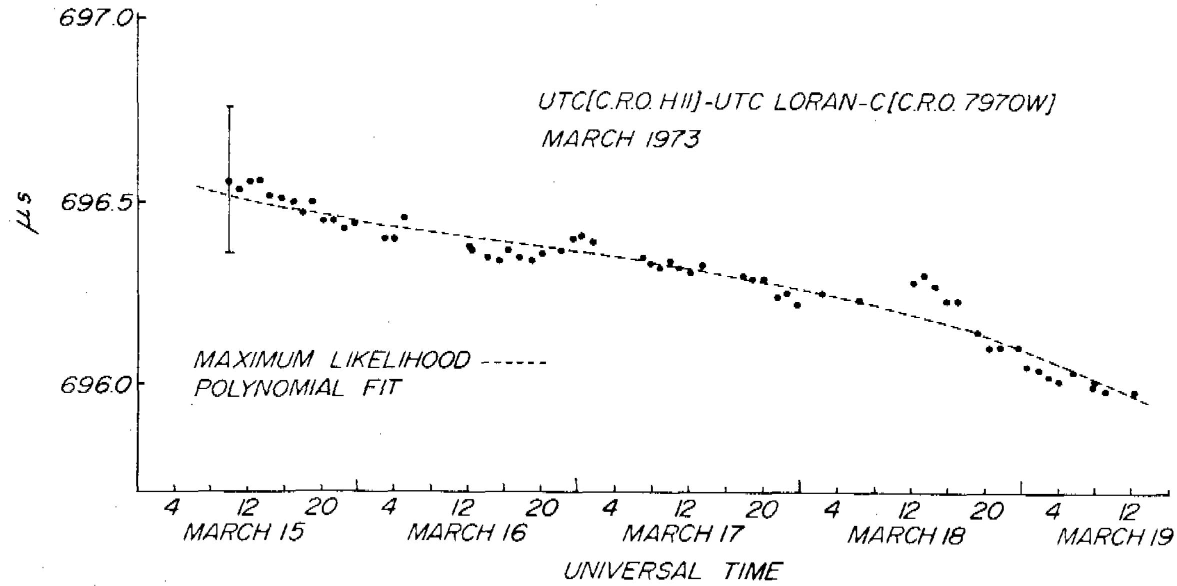

FIGURE 2b

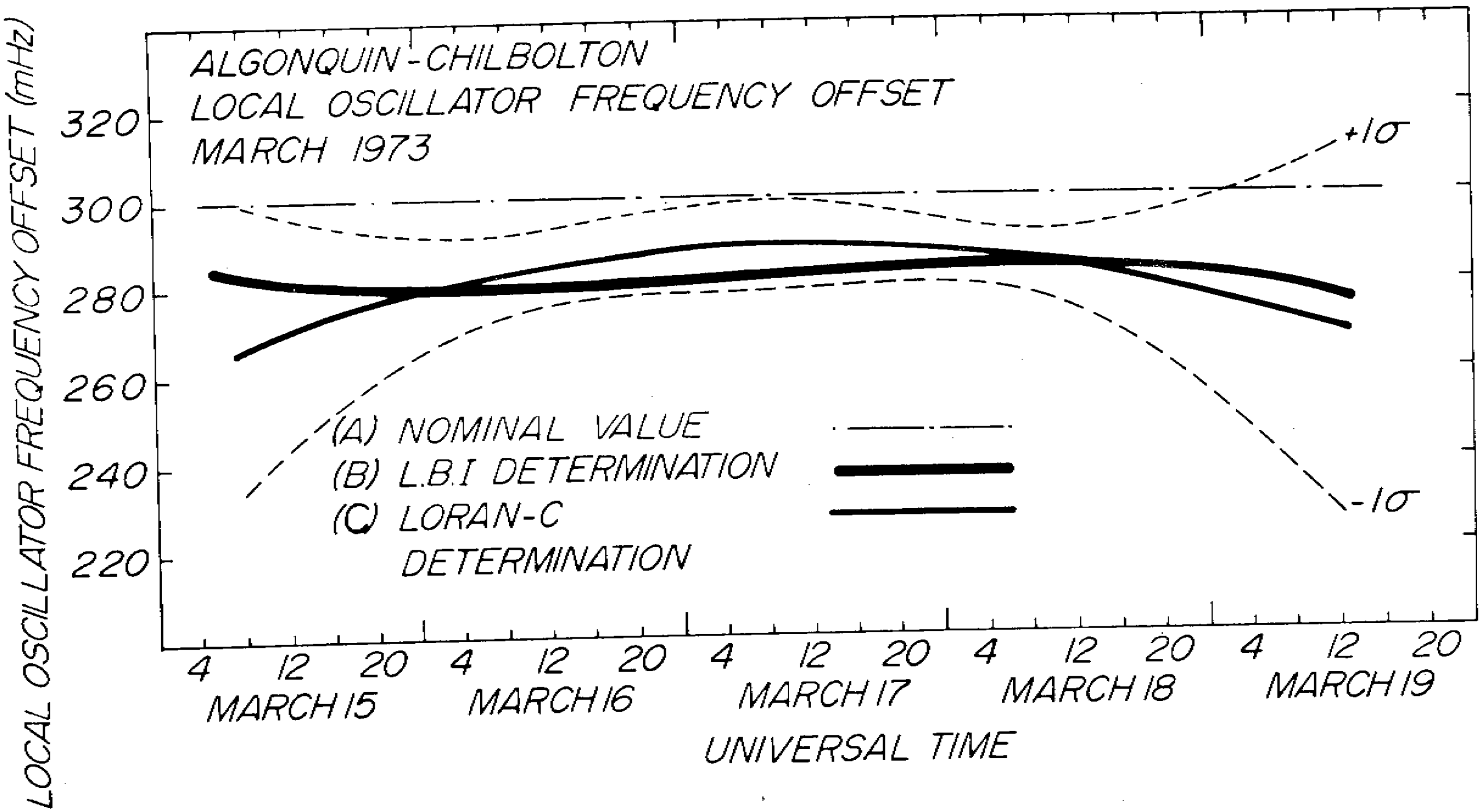

FIGURE 3. Long baseline interferometer local oscillator offset, $\Delta f_o(t)$, as determined by (a) Nominal Setting (b) LBI fringe frequency analysis (c) Loran-C/Time Service data. March 1973.

of the order of a few milliHertz as indicated by Robertson (1975). Also shown in FIGURE 3 are the $\pm$ 1σ standard deviations on $\Delta f_{o_{Loran}}(t)$ indicated by dashed lines and computed by the usual techniques of propagation of error assuming $\pm$ 0.2 μs standard deviation on the individual Loran-C timing measurements. The width of the $\pm$ 1σ region surrounding $\Delta f_{o_{Loran}}(t)$ should probably be somewhat larger as the authors have subsequently been informed (Lavanceau,1976) that the standard deviation on Loran-C timing measurements for the Norwegian Sea Chain is more typically $\pm$ 0.5 μs. In spite of this it is seen that the two methods of estimating the local oscillator offset agree rather well for the experiment of March 1973. Both estimates are seen from FIGURE 3 to disagree with the "expected" or nominal local oscillator offset of 300 mHz by about 20 mHz r.m.s. The disagreement of 20 mHz r.m.s. in 10 GHz between the two independent determinations of $\Delta f_o(t)$ on the one hand and the nominal value of $\Delta f_o(t)$ on the other hand is of the order of 2 parts in 10^{12} and is of the order of the reproducibility or "setability" of "commerical" hydrogen maser frequency standards (Audoin and Vanier, 1976). The disagreement between the two independent determinations of $\Delta f_o(t)$ by Loran-C on the one hand and by LBI on the other hand is of the order of 5 mHz r.m.s. or of the order of 5 parts in 10^{13} and is comparable to the reproducibility or setability of "laboratory" Cs beam frequency standards (Audoin and Vanier, 1976).

III JUNE 1973 EXPERIMENTAL RESULTS

The data for the experiment of June 1973 were much poorer than those of March 1973. In the first place due to a receiver malfunction at Chilbolton it was not possible to monitor the drift of the station clock, UTC[CRO-H11], against UTC Loran-C[CRO 7970 W]. Instead, a transfer frequency standard, Rb-340, was transported from the National Physical Laboratory (NPL) Teddington and the time difference UTC[CRO-H11] - UTC[Rb-340] was logged at Chilbolton during the experiment. At Algonquin Park the Loran receiver was functioning and the time difference UTC[ARO-H9]-UTC Loran-C[ARO 9930Y] was logged as in March 1973. Trouble with the local oscillator phase lock loop, once at Algonquin Park (∿0930 U.T. June 17, 1973) and twice at Chilbolton (0430 U.T. and 1300 U.T. June 17, 1973) produced large discontinuities in these two data sets as is indicated by FIGURES 4a and 4b. An attempt was made to estimate the magnitude of the "time glitches" by piecewise fitting curves to the continuous segments of the data and matching their first derivatives at the times of local oscillator phase lock failure. The resulting "deglitched" clock data are shown in FIGURES 5a and 5b.

In order to obtain a measure of the drift of the station clocks relative to each other, $\Delta\tau_{ARO/CRO}(t)$, it was necessary to refer UTC[CRO H11] back to UTC[USNO M.C.] in spite of the absence of the Loran-C timing data. This was accomplished through the use of the transfer standard Rb-340

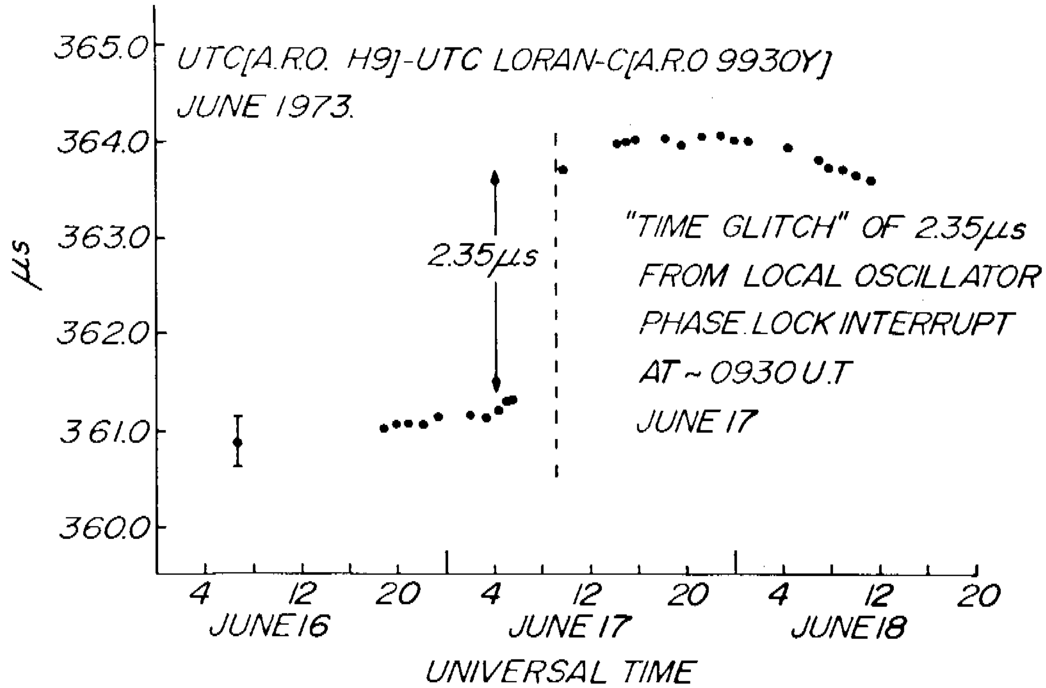

FIGURE 4a

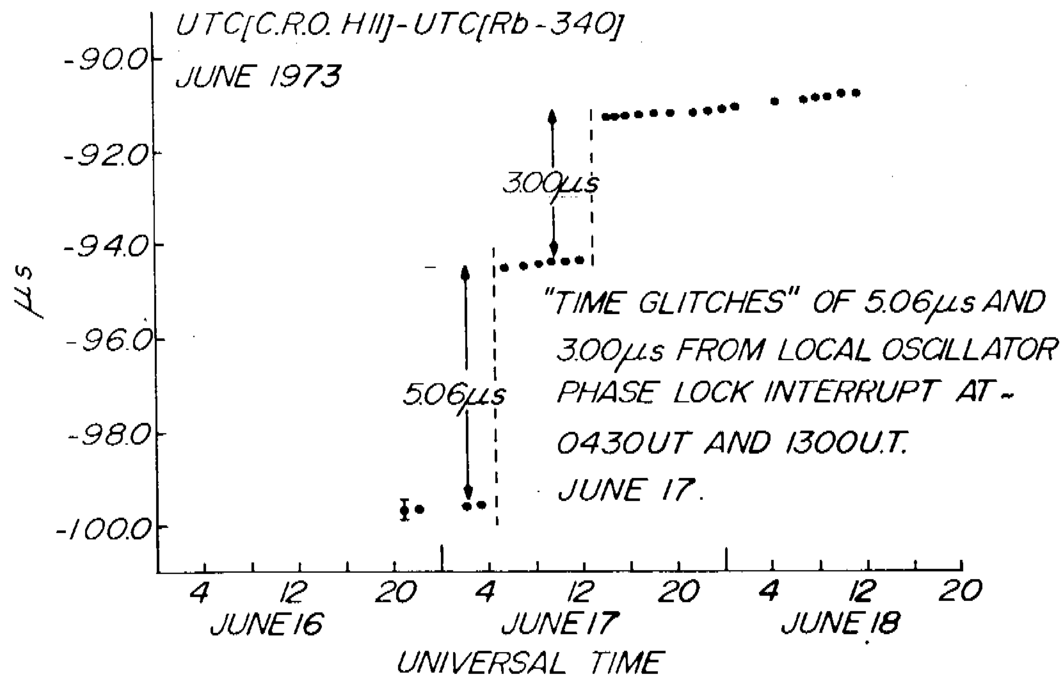

FIGURE 4b

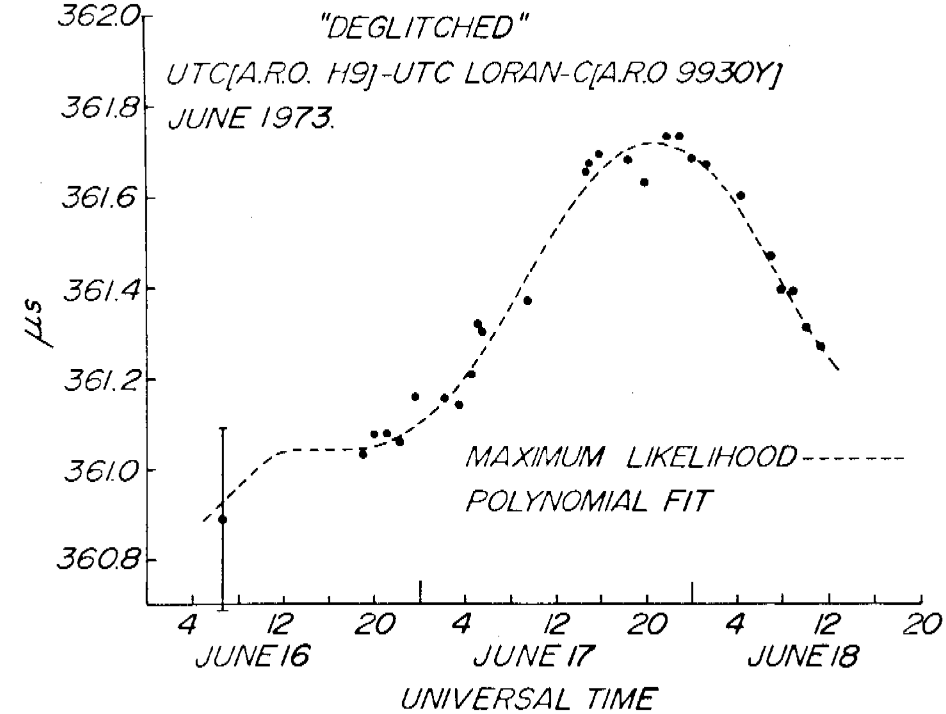

FIGURE 5a

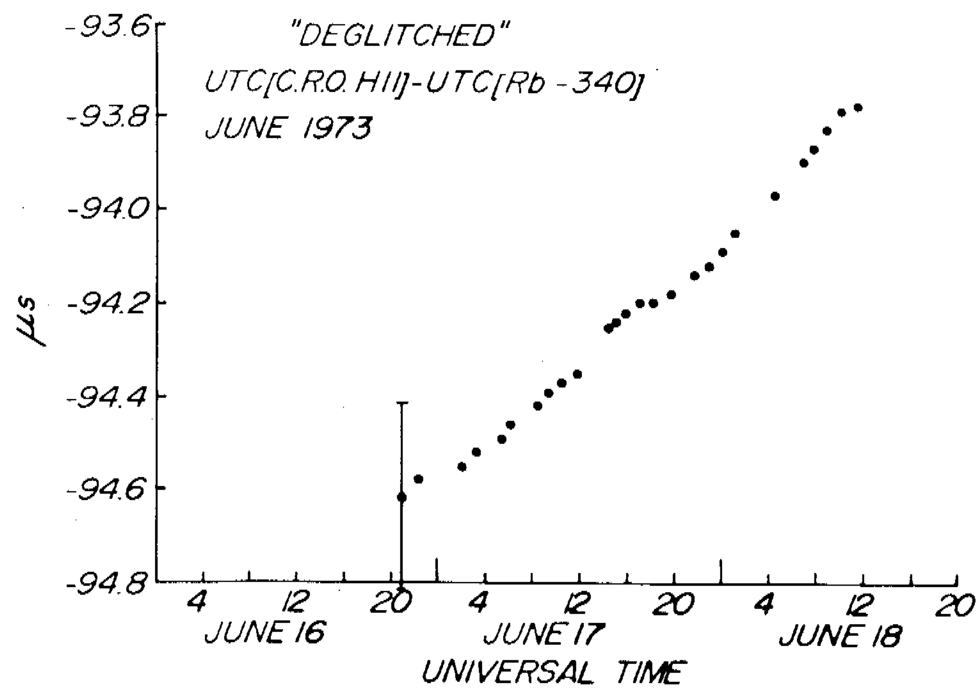

FIGURE 5b

and the laboratory frequency standard Cs-418 at NPL which provides time scale UTC[NPL]. The difference UTC[USNO-M.C.] - UTC [NPL] at ten day intervals, was then obtained from the Annual Report of the BIH (1974).

A comparison of the frequency standard Rb-340 against Cs-418 yielded

(i) ∿10:00 U.T. June 12 1973
UTC[NPL] - UTC[Rb-340] = 43.31 μs

(ii) ∿10:00 U.T. June 19 1973
UTC[NPL] - UTC[Rb-340] = 48.44 μs

It was necessary to assume that Rb-340 had drifted linearly relative to Cs-340 had drifted linearly relative to Cs-418 at the constant rate of -0.733 μs per day for the duration of the experiment. With this assumption the drift of the "station clock" UTC[CRO Hil] relative to UTC[NPL] could be established. The results of this calculation together with a maximum likelihood polynomial fit to the data are shown in FIGURE 6.

The rate of Cs-418 relative to USNO M.C. was established by a linear fit to the BIH data for UTC[USNO M.C.] - UTC[NPL] obtained from the

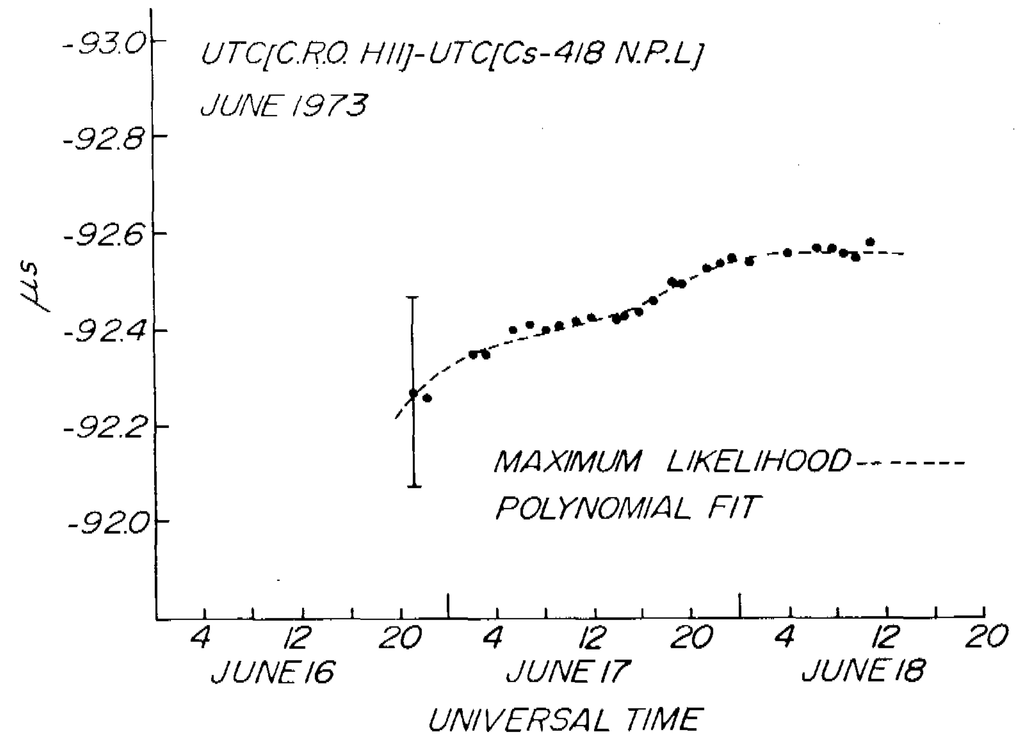

FIGURE 6

BIH Annual Report for 1973 pp 30-33 (1974). Since NPL readjusted the C field, beam current, and loop gain on Cs-418 on June 25, 1973 (Swabey, 1976) the rate of UTC[NPL] relative to UTC[USNO M.C.] for the June experiment was obtained on the basis of BIH data covering the interval January 7, 1973 - June 16, 1973. During this interval UTC [USNO M.C.] - UTC[NPL] was changing at the rate of -0.050 μs per day.

The results of this analysis are shown in FIGURE 7. The heavy curve indicates $\Delta f_{o_{LBI}}(t)$, the LBI determination of the local oscillator frequency offset including small relativistic corrections (Robertson, 1976) and the lighter curve indicates $\Delta f_{o_{Loran}}(t)$, the Loran-C/Time Service determination of the local oscillator frequency offset. The disagreement between the two solutions is quite large, ∿60 mHz r.m.s., or roughly 6 parts in 10^{12} compared to the results for the March experiment which were ∿5mHz r.m.s. or 5 parts in 10^{13} respectively.

The experimental procedure for June 1973 was clearly inferior to that of March 1973. The experimental procedure for March 1973 allowed (subject to the limitations on precision inherent in Loran-C timing measurements) a more or less continuous, independent monitoring of the relative rate of the two station clocks for comparison with the LBI solution. This was not the case in the June 1973 experiment which required interpolation of clock rates over 5 and 10 day intervals. Considering the large number of assumptions and corrections (each of them with their attendant errors) required to analyse the June 1973 data, even these somewhat poorer results should perhaps be seen as encouraging.

Again in June 1973 as in March 1973 the two independant solutions to the local oscillator frequency offset disagree with the nominal value of 500 mHz by significantly greater amounts than that by which they disagree with each other.

IV CONCLUSIONS

This study appears to indicate that the technique of LBI compares favourably with the use of the Loran-C/Time Service method as a means of monitoring frequency differences between remotely sited frequency standards.

The practical limitations on the precision with which this can be achieved by the Loran-C/Time Service technique are imposed by the variations in the propagation delays of the Loran-C pulses which are the order of ± 0.2 μs - ± 1.0 μs. Even assuming favourable conditions this effect appears to produce a ± 1σ standard deviation of the order of 10-20 mHz relative oscillator frequency difference for oscillators whose frequencies are in the 10 GHz range. Thus it would appear that the Loran-C/Time Service technique allows a comparison of remote

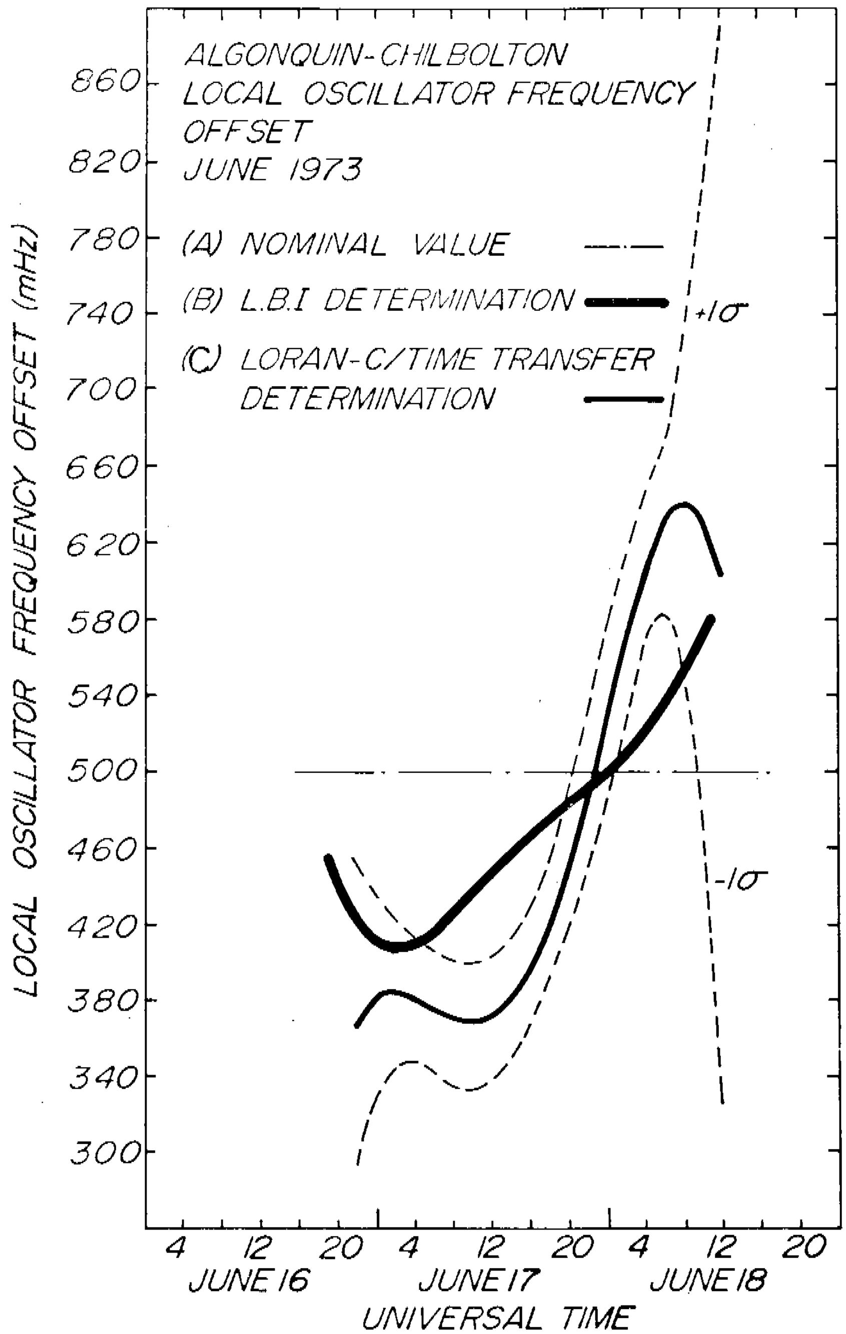

FIGURE 7. Long baseline interferometer local oscillator offset, $\Delta f_o(t)$, as determined by (a) Nominal setting (b) LBI fringe frequency analysis (c) Loran-C/Time Service data June 1973.

frequency standards with a maximum precision of roughly one part in 10^{12} for ground wave Loran-C reception averaged over a few days.

The practical limitations on the LBI technique are imposed by:

(a) uncertainties in the baseline length and orientation

(b) uncertainties in the source coordinates

(c) uncertainties in the atmospheric transmission effects on the received RF signal

(d) uncertainties in the short term rotation rate of the earth.

Estimates of the magnitudes of these effects can be made by combining typical values of the partial derivative of fringe frequency with respect to these various parameters with typical values of their uncertainties. For an interferometer baseline $\sim$5000 km and observing frequency $\sim$10 GHz these calculations are summarized in TABLE I.

TABLE 1.

Uncertain quantity	Typical uncertainty	Typical contribution to fringe frequency
vector baseline	$\pm$ 1 m. each coordinate	$\pm$ 1 mHz
source coordinates	$\pm$ $0''.1$ each coordinate	$\pm$ 3 mHz
atmospheric phase delay rate	phase delay fluctuations $\sim$6 cm. Time scale $\sim$15 minutes	$\pm$ 2 mHz
short term rotation rate of earth	l.o.d. known to $\pm$ 3 ms on a single night	$\pm$ 3 mHz

These various effects all combine to produce an uncertainty of $\pm$ 5 mHz in the LBI determination of the relative frequency of remotely sited oscillators in the 10 GHz range. This corresponds to a precision of roughly five parts in 10^{13} or comparable to the precision of the Loran-C/Time Service technique. This is confirmed by the experimental results reported here.

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